Pedro Tanure Veloso

Introducing Capital in a Three-Sector Model of Structural Transformation:

A Cross-Country Analysis

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Co–Advisor: Prof. Carlos Viana de Carvalho

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Resumo

Veloso, Pedro Tanure; Berriel, Tiago Couto; Viana de Carvalho, Carlos. **Introducing Capital in a Three-Sector Model of Structural Transformation: A Cross-Country Analysis**. Rio de Janeiro, 2015. 58p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

A literatura recente tem enfatizado a importância do crescimento da produtividade setorial em explicar a realocação de emprego entre setores ao longo do tempo–fenômeno conhecido como transformação estrutural. Desenvolvemos um modelo de transformação estrutural com capital, permitindo que o investimento seja feito por múltiplos setores. Avaliamos os efeitos do crescimento de produtividade setorial, mudanças nas participações setoriais no investimento, e crescimento no estoque de capital em explicar transformação estrutural e produtividade agregada. Encontramos que crescimento da produtividade da agricultura foi importante para explicar a realocação setorial do trabalho, mas não o crescimento da produtividade na manufatura e nos serviços. Estes resultados contrastam com o que foi ultimamente explorado pela literatura usando um arcabouço simples sem capital. Acumulação de capital tem um papel importante na transformação estrutural. Defendemos que considerar um arcabouço mais simples pode levar a conclusões falsas sobre quais são os principais fatores que afetam o processo de transformação estrutural e produtividade agregada entre países.

Palavras–chave

Transformação Estrutural; Acumulação de Capital; Produtividade Setorial;

Abstract

Veloso, Pedro Tanure; Berriel, Tiago Couto; Viana de Carvalho, Carlos. **Introducing Capital in a Three-Sector Model of Structural Transformation: A Cross-Country Analysis**. Rio de Janeiro, 2015. 58p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Recent literature has highlighted the importance of sectoral productivity growth in explaining reallocation of employment across sectors through time–the so-called structural transformation. We develop a model of structural transformation with capital, allowing for multiple-sector investment. We assess the effects of sector-specific productivity growth, changes in sectoral investment shares, and growth in capital stock in explaining structural transformation and aggregate productivity. We find that sectoral productivity growth in agriculture was important in explaining sectoral labor reallocation, but not productivity growth in manufacturing and services. These results contrast with what was recently explored in the literature using a simple framework without capital. Capital accumulation actually plays an important role in explaining structural transformation. We argue that considering a simpler framework can lead to false conclusions about what are the main drivers of structural transformation and aggregate productivity across countries.

Keywords

Structural Transformation; Capital Accumulation; Sectoral Productivity;

Contents

List of figures

List of tables

1
Introduction

Structural transformation has been the subject of intense research, particularly in the last twenty years. In particular, some studies explore the link between sectoral productivity growth and reallocation of the labor force across sectors and also how they affect aggregate productivity, measured as output per hour worked. Duarte and Restuccia (2010) studied how different growth rates of sectoral labor productivity (value added per hour worked) affected the process of structural transformation, and how these two effects combined affected aggregate productivity across a large set of countries. The authors build and calibrate a three-sector model where labor is the only input of the firms and use it to measure sectoral labor productivity across countries. They find that productivity catch-up in manufacturing relative to the United States was the key driver of the gains in aggregate productivity. When labor is reallocated from agriculture to manufacturing, this shift to a more productive sector improves aggregate productivity. However, as economies mature and labor moves to the service sector, poor catch-up of productivity in services is responsible for the decline, stagnation or slowdown observed in the data.

When treating labor as the only input for sectoral technology, Duarte and Restuccia (2010) abstract from the effects of capital accumulation in the process of structural transformation. In fact, as will be shown below, capital accumulation can make labor shift from one sector to another, even if capital's share on production is equal across sectors. Part of the reason is due to the income effects of capital accumulation. Therefore, it is important to investigate the role of capital accumulation combined with sectoral labor productivity growth in the process of structural transformation.

When analyzing sectoral labor reallocation through the lens of a model with capital accumulation, studies usually either assume that manufacturing is responsible for the investment in the economy or that the economy is composed of only two sectors: agriculture and non-agriculture. The first approach is usually flawed: manufacturing tends to be smaller than investment as share of GDP (Herrendorf et al., 2014). Additionally, as we will see below, sectoral composition of investment is neither entirely concentrated in the manufacturing sector nor it is stable across time. Although the second approach mitigates the problem faced by the manufacturing assumption, it can reduce the framework too much if one is interested in a three-sector analysis of structural transformation, as we are.

This paper builds upon the work of Duarte and Restuccia (2010) and analyzes the process of structural transformation in a model that uses capital as input in technology across sectors. Departing from the literature, we build a three-sector growth model with capital accumulation where we will allow for investment by agriculture, manufacturing and services. This is possible due to data available from the national accounting tables from the United States, calculated by Herrendorf et al. (2013). Here, sectoral investment shares are treated as exogenous and used in the calibration. Assuming that capital share is equal across sectors, we are able to write sectoral production functions as a function of aggregate capital. This also enables us to aggregate output and derive the series of capital stock which, combined with sectoral productivity growth and the share of investment made by each sector, solves the equilibrium system of equations that pins down sectoral employment shares.

After developing the three-sector model, we start by using the United States as a benchmark economy for our quantitative exercise. We calibrate the model and check its solution. The model is able to replicate all the salient features of growth and structural transformation in the United States from the 1950's to 2005, such as employment shares, sectoral value added, capital stock and changes in relative prices. Combining data on capital stock from a panel of countries relative to the one in the United States, we then use the framework from Duarte and Restuccia (2010) to pin down relative sectoral productivity in the first period of the sample using our framework. This enables us to repeat the same exercise to our panel of countries. We find that the performance of the model is close to the predictions made by Duarte and Restuccia's framework, especially in developed countries, being able to replicate well salient features of share of hours worked by sector[1] and relative aggregate productivity.

Narrowing down the analysis to this group of countries, we perform a series of counterfactual exercises to asses the importance of each component driving sectoral dynamics in our model: sectoral productivity growth and capital accumulation. We find that sectoral productivity growth, especially in agriculture, is an important driver of employment out of agriculture to manufacturing and services. In contrast to Duarte and Restuccia's framework, productivity growth in the manufacturing sector plays a smaller role in sectoral labor reallocation. A catch-up in productivity growth in manufacturing relative to the United States in our model predicts a large increase in aggregate relative productivity. Due to the difference in the results from both frameworks, we then move on to analyze the role of capital accumulation. We assess if results from the counterfactual exercises using the authors' framework were due to

[1]In this work, we will use the terms employment and hours worked interchangeably.

the fact that their measure of sectoral productivity, output per hour worked, was too influenced by the capital stock embedded in such measure. Assuming that capital remains constant in the first period of out analysis and allow for sectoral productivity growth, we find that this has relative little effect on aggregate relative productivity, but it substantially curbs sectoral employment reallocation. This result is in line with what we expected: using output per hour as a measure of sectoral productivity inevitably mixes effects of sectoral productivity growth and capital accumulation, which we believe should be properly separated and understood. Finally, as a robustness exercise, we find that different assumptions on sectoral investment can bias the labor share predicted by the model. Notably, assuming that manufacturing is the only sector capable of performing capital accumulation biases equilibrium labor shares significantly in favor of manufacturing employment.

These findings shed an important light on the debate about the effects of sectoral productivity on structural transformation and aggregate productivity. In contrast to previous literature, our conterfactuals suggest that sectoral productivity growth in manufacturing and services in fact play a much smaller role in explaining sectoral labor reallocation. In addition, they also indicate that the role of productivity growth in services is less relevant with respect to its impact on aggregate productivity. These findings bring us to believe that using a more complete model of structural transformation, including capital as an input for output production, is important to single out the effects of productivity growth and capital accumulation and the role each factor plays in the process of labor reallocation.

Such results also affect how one thinks about policy prescriptions. Since we take into account capital in sectoral output per hour, our measure of sectoral productivity is more strict and behaves differently from the one we find in a framework without capital. Even though market equilibrium is efficient in both frameworks, they don't explicitly model the behavior of the productivity component of sectoral output.[2] When thinking if policies should induce or boost productivity of an specific sector or the economy as a whole, one has to think if such policies are not misguided, if in contrast efforts should be directed to increasing the economy's ability to invest and accumulate capital and to increasing overall productivity of workers. This question cannot be answered by a simpler framework than ours simply because considering value added per hour worked as a measure of labor productivity and performing counterfactuals to understand the behavior of the economy will ignore the role capital plays in

[2]Therefore, the policy issues we are concerned here are the ones affecting such measures of productivity.

output and employment in these sectors. Therefore, we believe that considering a stricter measure of sectoral productivity while understanding the effects of investment on structural transformation is crucial in such debates.

1.1 Structural Transformation: A Bird's Eye View

Over the past century, economic growth and development across countries has been pervasive. Most countries have experienced a large improvement in the living conditions and significant rise in disposable income of their citizens. Additionally, the amount of output generated by the average worker has increased substantially.

Besides the increase in output per hour worked, another important feature of the last century has been the gradual change in the sectoral composition of the output produced. As noted by Kuznets (1973), the process of economic development is accompanied by a shift in the share of both the output and employment in different sectors of the economy. When countries are relatively poor, most of the labor force is concentrated in the agricultural sector of the economy. The share of output produced by this sector, likewise, is also considerable. In the first stage of economic development, when countries escape the poverty trap, labor tends to be reallocated from agriculture to the manufacturing sector of the economy, hence increasing its share in total output produced. In the second and final stage of development, labor is reallocated from both manufacturing and agriculture to the service sector. That is, the process of economic development also has the effect of shifting labor and output shares from agriculture to services, with an intermediate step in which manufacturing rises and subsequently reduces its relative share in the economy. This process is usually referred in the literature as either structural transformation or structural change.

This pattern is clear in available data for different countries. Here, we use time series on sectoral employment and value added for Latin America, Asia and OECD countries, available from the 10-Sector Database. Data is available from the 1950's to early 2010's. Figure 1.1 plots the historical time series. The horizontal axis reports the log of GDP per capita in 2005 PPP dollars, available from the Penn World Tables. The vertical axis reports sectoral shares of employment and value added in current prices. The figures show what has been documented as the *Kuznets facts*: agricultural share on employment and value added constantly decline as GDP per capita grows, while the opposite happens in the service sector. Manufacturing, however, displays a hump-shaped pattern: its shares increase on early stages of development, while diminishing

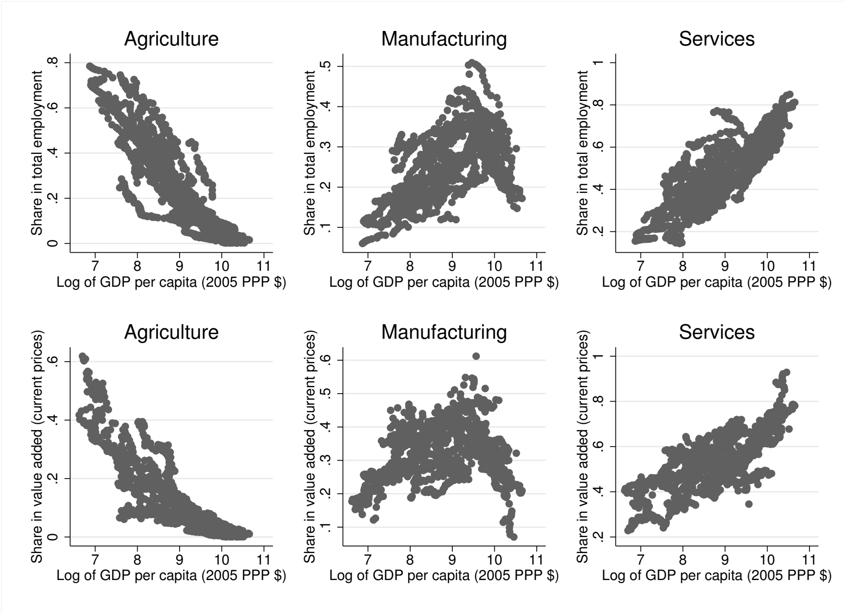

Figure 1.1: Sectoral Shares of Employment and Value Added

as the economies continue to develop.

What Figure 1.1 also shows is that, in early stages of development, value added from agriculture is less than its counterpart in employment, while the opposite is true at services. Additionally, manufacturing shares seem to peak when the log of GDP per capita reaches around nine. After that, shares from the service sector seem to increase more rapidly. This coincidence suggest that the service sector is partly responsible for the decline in manufacturing shares, which is exactly what Kuznets (1973) has argued.

1.2 Theoretical Analysis of Structural Transformation: Key Results

In the growth literature, structural transformation and sectoral composition of employment and production are usually neglected, since theoretical models tend to aggregate consumption and production into one sector. If one is interested in for different sectors in the economy and structural transformation, the concept of balanced growth path, in which all endogenous variables grow at the same rate, is too strict. Sectoral composition changes along structural transformation. The literature has managed this issue by applying a less strict concept of *generalized balanced growth path* (GBGP), that only requires that the real interest rate is constant.

Kongsamut et al. (2001) developed a special case that combines structural transformation and GBGP. In their setting, labor-augmenting technical progress is the same rate across sectors. Additionally, per-period utility function with Stone-Geary preferences takes the following form:

$$C_t \equiv \left(A_t - \bar{A}\right)^{\beta} M_t^{\gamma} \left(S_t + \bar{S}\right)^{\theta},$$

where β, γ, θ, $\bar{A}$, $\bar{S}$ are all positive constants, A_t is the consumption of agricultural goods, M_t is the consumption of manufactured goods, and S_t is the consumption of service goods. The reason why the authors chose this functional form is that they wanted to analyze structural transformation based only on *income effects*. The transmission channel can be summarized as follows: the term $\overline{A}$ is equivalent to a subsistence level of agricultural goods, below which the household cannot survive. Similarly, $\overline{S}$ can be interpreted as a negative subsistence level, or as a reduced form for domestic production of services. When the utility function takes the form as described above, as household's income rises, more resources are allocated to service goods, and less for agricultural goods. Thus, as the country grows richer, structural transformation emerges as a result of household's preferences.

The production side of the economy comprises three sectors, where manufacturing accumulates capital, which is a common assumption in this literature. They show that a GBGP exists and is unique if some parameters of their model have very specific values. Although the existence and uniqueness of the GBGP requires somewhat strict assumptions, they developed a model that can be solved analytically.[3]

Ngai and Pissarides (2007) developed another special case, focusing on *relative price effects* of the structural transformation. In their model, per-period utility function has constant elasticity of substitution for m sectors in the economy:

$$C_t \equiv \left(\sum_{i=1}^{m} \omega_i c_{it}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

The supply side of their model assumes that $m-1$ sectors produce consumption goods, but sector m production may be either consumed or invested. The key assumption from Ngai and Pissarides (2007) is that sectoral productivity grow at different rates, therefore implying different paths for relative prices. The authors derive two core results. The first one is that a logarithmic lifetime utility and productivity growth rate of at least one sector different from m are necessary and sufficient conditions for the existence of a GBGP. The second result is that, if $\epsilon < 1$ and sectoral productivity growth varies across sectors, the sector with slower productivity growth will feature

[3]Echevarria (1997) presents a different model that emphasizes income effects of structural transformation, but uses different utility specification such that an interior solution to household's problem exists for any level of income.

monotonic increase in its employment share, while employment in other sectors is either hump-shaped or declines monotonically. Under such conditions, the GBGP is unique.

1.3 Empirical Findings of the Literature

There is also an extensive literature studying the empirical effects of structural transformation on economic development and growth. Here, we will focus on those who emphasized the role of sectoral productivity, income and relative price effects.

Gollin et al. (2002) study how the early process of economic development and industrialization was affected by sectoral productivity. They use a two-sector model (agriculture and non-agriculture) with Stone-Geary preferences to analyze the role of agricultural productivity in explaining both the timing and the speed of industrialization. They find that higher productivity growth in agriculture is crucial for the onset of industrialization and also accelerates the process of structural transformation.

Dennis and Iscan (2009) investigate the importance of sectoral productivity, income effects and capital deepening in two hundred years of economic development in the United States. Their model features two sectors, agriculture and non-agriculture, with different capital shares across sectors. They also allow for the non-homothetic term in agriculture to vary across time. They calibrate their model and estimate the time path of labor supply in agriculture as a function of structural parameters and sectoral productivity. Their model results are broadly consistent with U.S. data, especially when they consider a time trend in the non-homothetic parameter of agriculture. They find that, prior to 1950, income effects are the most important factor driving structural transformation. After 1950, capital deepening and relative price effects play a larger role in explaining labor reallocation, although this role is still not as relevant as the one played by income effects.

Since developed countries have already experienced the process of structural transformation, some research is directed to understand how labor is allocated though times in emerging economies. Dekle and Vandenbroucke (2012) study how the gradual reduction of the government size affected China's structural transformation. The authors develop a three-sector model, that includes agriculture, non-agriculture and government. In their model, the share of labor allocated to government sector is treated as a tax to household's labor supply. The production of agricultural goods also use a fixed stock of land. They include a wedge between agricultural and non-agricultural wages to reflect the fact that migration from rural to urban areas is largely discouraged by the Chinese government. Their study finds that the most important force driving China's structural transformation is the productivity growth in agricultural sector. The government "tax" on household employment and mobility costs also have important effects on sectoral labor allocation, since they account for almost thirty-five percent of labor reallocation.

Structural transformation can also be affected by international trade. Declines in trade costs and different sectoral productivity growth both affect patterns of specialization, which in turn affect labor allocations. Finaly, lower trade costs stimulates income growth and thus cause structural transformation because of income effects. Authors such as Uy et al. (2013) study the effect of international trade in structural transformation focusing their empirical study on South Korea. They find that higher growth in productivity in manufacturing and decreasing trade cost in this sector both caused trade specialization and labor reallocation to manufacturing. When compared to a closed-economy framework, they find that trade specialization is an important factor of labor reallocation from agriculture to manufacturing.

Duarte and Restuccia (2010) study the role of different paths of sectoral productivity growth in explaining the process of structural transformation and aggregate productivity. They use a three-sector model, mixing Stone-Geary and CES preferences, along with non-homotheticicities in agriculture and services, while modeling the supply side using labor as the only input. The authors first calibrate their model to match the structural transformation of the United States from 1956 to 2004, normalizing sectoral productivity to one in 1956, and using the growth rate of real value added per hour available from data to create time paths of productivity growth.

The model is then used to measure labor productivity across 29 countries relative to the United States for the period analyzed. They find large productivity differences in agriculture and services, while the gap is smaller in manufacturing. Over time, productivity catches up, especially in manufacturing and agriculture, while services remain largely less productive in other economies. Their model also broadly explains structural transformation across countries in their sample. During the process, as labor reallocates from agriculture to manufacturing, aggregate productivity catches up relative to the United States. When labor shifts from manufacturing to services, poor labor productivity in the latter leads to a falling behind of aggregate productivity relative to the United States. The core result of their study is that structural transformation, combined with low levels of productivity in services, play a key role in determining whether a country's aggregate productivity growth declines, stagnates or decline over time.

Since Duarte and Restuccia (2010) abstract from effects of capital accumulation in their model, their results mix effects of capital stock and sectoral productivity growth. As argued above, although their results illustrate the effect of labor productivity growth in structural transformation and the time path of aggregate productivity, we believe that separating effects of capital accumulation and productivity can improve our understanding of how structural transformation and aggregate economic performance are related.

2
A Model of Structural Transformation

To understand how capital accumulation and labor productivity drive the process of structural transformation, we build a three-sector model with capital, keeping the same structure for the representative agent as in Duarte and Restuccia (2010). Time is discrete and indexed by $t = 0, \ldots, \infty$. There is a single infinitely lived representative household that discounts time at rate $\beta \in (0, 1)$ and is endowed with one unit of labor per period and an initial stock of capital K_0. Physical capital depreciates at rate δ and the interest rate between period $t-1$ and t is denoted by R_t. The real wage during period t is denoted by W_t.

2.1
Household

The representative household has Stone-Geary preferences between agricultural goods, c_{at}, and non-agricultural goods, c_{nat}, at each date. This feature adds a non-homotheticity in the utility function, with elasticity of substitution among consumption goods equal to one. Aggregate consumption index is defined as

$$C_t \equiv (c_{at} - \bar{a})^a (c_{nat})^{1-a},\ a \in [0, 1],\ \bar{a} > 0.$$

The parameter $\bar{a}$ follows Kongsamut et al. (2001), being equivalent to a subsistence level of agricultural goods. For non-agricultural goods, we have a CES aggregator for manufacturing, c_{mt}, and services, c_{st}:

$$c_{nat} \equiv [bc_{mt}^{\rho} + (1-b)(c_{st} + \bar{s})^{\rho}]^{\frac{1}{\rho}},\ b \in (0, 1),\ \bar{s} > 0,\ \rho < 1.$$

In this setting, $\bar{s}$ can be interpreted as a constant level of production of service goods at home. Parameter ρ is the elasticity os substitution between manufacturing an service goods. Note that $\bar{s}$ implies that the income elasticity of service goods is greater than one. We will also assume a logarithmic functional form for lifetime utility function.

In our model, the household chooses between consumption among goods in current period and aggregate capital one period ahead, K_{t+1}. Labor is inelastically supplied. Household's problem therefore can be written as:

$$\max_{\{c_{at}, c_{mt}, c_{st}, K_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left\{ a \log(c_{at} - \bar{a}) + (1-a)\frac{1}{\rho} \log[bc_{mt}^{\rho} + (1-b)(c_{st} + \bar{s})^{\rho}] \right\}$$

$$s.t. \quad p_{at}c_{at} + p_{mt}c_{mt} + p_{st}c_{st} + K_{t+1} = (1 - \delta + R_t)K_t + W_t.$$

Let λ_t be the Lagrange multiplier. Assuming interior solution,[1] first order conditions for the consumption goods are:

$$\frac{a}{c_{at} - \bar{a}} = \lambda_t p_{at} \tag{2-1}$$

$$\frac{1-a}{c_{nt}^{\rho}} b c_{mt}^{\rho-1} = \lambda_t p_{mt} \tag{2-2}$$

$$\frac{1-a}{c_{nt}^{\rho}} (1-b)(c_{ct} + \bar{s})^{\rho-1} = \lambda_t p_{st}, \tag{2-3}$$

which can be rewritten as:

$$a = \lambda_t p_{at}(c_{at} - \bar{a})$$

$$\frac{1-a}{c_{nt}^{\rho}} b c_{mt}^{\rho} = \lambda_t p_{mt} c_{mt}$$

$$\frac{1-a}{c_{nt}^{\rho}} (1-b)(c_{ct} + \bar{s})^{\rho} = \lambda_t p_{st}(c_{ct} + \bar{s}).$$

Adding the rewritten equations (2-1)-(2-3), we obtain

$$a + \frac{1-a}{c_{nt}^{\rho}} \underbrace{[b c_{mt}^{\rho} + (1-b)(c_{st} + \bar{s})^{\rho}]}_{c_{nt}^{\rho}} = \lambda_t [p_{at}(c_{at} - \bar{a}) + p_{mt}c_{mt} + p_{st}(c_{st} + \bar{s})],$$

which implies

$$\lambda_t^{-1} = p_{at}(c_{at} - \bar{a}) + p_{mt}c_{mt} + p_{st}(c_{st} + \bar{s}) \equiv P_t C_t. \tag{2-4}$$

Given that λ^{-1} is the marginal value of one additional unit of expenditure in period t, we can define this value as the product of total consumption level, C_t, and an aggregate price level, P_t. Hence, we can write:

$$p_{at}c_{at} + p_{mt}c_{mt} + p_{st}c_{st} = P_t C_t + p_{at}\bar{a} - p_{st}\bar{s},$$

where P_t is defined as:[2]

$$P_t \equiv \left(\frac{p_{at}}{a}\right)^{a} \left\{ \frac{\left[b^{\frac{1}{1-\rho}} (p_{mt})^{\frac{\rho}{\rho-1}} + (1-b)^{\frac{1}{1-\rho}} (p_{st})^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}}{1-a} \right\}^{1-a}.$$

It follows that household's original problem can be split into two sub-

[1] In fact, non-homothetic utility functions can lead to corner solutions. However, this issue only seems relevant in very poor countries, where structural transformation is still on its early stages. Such countries are not the focus of our research.

[2] See Appendix A for a formal derivation of the price index.

problems:

1. **Intertemporal Problem.** Allocate income among consumption index and savings:
$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t$$
$$s.t. \quad P_t C_t + K_{t+1} = (1-\delta+R_t)K_t + W_t - p_{at}\bar{a} + p_{st}\bar{s}.$$

2. **Static Problem.** Allocate consumption expenditure between different consumption goods:
$$\max_{c_{at}, c_{mt}, c_{st}} (c_{at}-\bar{a})^a \{[bc_{mt}^{\rho} + (1-b)(c_{st}+\bar{s})^{\rho}]^{\frac{1}{\rho}}\}^{(1-a)}$$
$$s.t. \qquad p_{at}c_{at} + p_{mt}c_{mt} + p_{st}c_{st} = P_t C_t + p_{at}\bar{a} - p_{st}\bar{s}.$$

This representation was proposed by Herrendorf et al. (2014). It separates the growth component from the structural transformation in the model. It is easy to see that the first subproblem is very similar to the one we find when studying one-sector growth models, now added with a time varying endowment $p_{at}\bar{a} - p_{st}\bar{s}$. From the perspective of structural transformation, the static problem is very similar to the one found in Duarte and Restuccia (2010).

We end this section by solving household's intertemporal problem. Let μ_t be the Lagrangian multiplier. First order conditions are:
$$\frac{\beta^t}{C_t} = \mu_t P_t$$
$$\mu_t = (1-\delta+R_{t+1})\mu_{t+1}$$
which gives us the following Euler equation:
$$\frac{1}{\beta}\frac{P_t C_t}{P_{t-1}C_{t-1}} = 1-\delta+R_t. \tag{2-5}$$
Additionally, the transversality condition is given by:
$$\lim_{t\to\infty} \beta^t \frac{K_t}{C_t} = 0. \tag{2-6}$$
Equations (2-5) and (2-6) are both necessary and sufficient conditions for the optimal consumption path.

2.2

Firms

We allow for each sector to produce a good that can be converted either into a specific consumption good or into capital one period ahead. The share of investment made by each sector is treated as exogenous and time-varying. Production functions are given by

$$y_{it} = k_{it}^{\alpha}(A_{it}n_{it})^{1-\alpha}, \qquad i \in \{a, m, s\},$$

where α is the capital share. The assumption that α is the same in all sectors is useful for tractability of our framework. Although it may seem odd at first glance, Gollin (2002) showed that aggregate capital shares variation is uncorrelated with per capita income across countries. Since the share of agriculture in GDP declines when income per capita increases and the opposite is observed in services, this means that capital shares don't differ much across sectors. Otherwise, aggregate labor shares would differ systematically between countries in different levels of development. Additionally, Herrendorf et al. (forthcoming) provide evidence for the postwar U.S. economy that differences in technical progress are the predominant force behind structural transformation and that assuming sectoral Cobb–Douglas production functions with equal capital shares captures the main trends of U.S. structural transformation.

First order conditions from the firm's problem are given by:

$$R_t = p_{it}\alpha\left(\frac{k_{it}}{n_{it}}\right)^{\alpha-1} A_{it}^{1-\alpha}, \qquad i \in \{a, m, s\} \tag{2-7}$$

$$W_t = p_{it}(1-\alpha)\left(\frac{k_{it}}{n_{it}}\right)^{\alpha} A_{it}^{-\alpha} \tag{2-8}$$

which implies that, for $i \in \{a, m, s\}$,

$$\frac{W_t}{R_t}\frac{\alpha}{1-\alpha} = \frac{k_{it}}{n_{it}}. \tag{2-9}$$

2.3 Market Clearing and Equilibrium

At every date, labor demanded from firms must equal the exogenous labor supply by the household:

$$1 = n_{at} + n_{mt} + n_{st}. \tag{2-10}$$

The demand for capital from firms must equal the aggregate stock of capital in the economy:

$$K_t = k_{at} + k_{mt} + k_{st}. \tag{2-11}$$

For $i \neq j$, equation (2-9) implies that[3]

$$\frac{k_{it}}{n_{it}} = \frac{k_{jt}}{n_{jt}} = K_t, \tag{2-12}$$

which means that production functions can be rewritten as

$$y_{it} = K_t^{\alpha} A_{it}^{1-\alpha} n_{it}, \quad i \in \{a, m, s\}. \tag{2-13}$$

From the household's intertemporal budget constraint, we can write:

$$P_t C_t + K_{t+1} - (1-\delta)K_t + p_{at}\bar{a} - p_{st}\bar{s} = p_{at}y_{at} + p_{mt}y_{mt} + p_{st}y_{st} \equiv Y_t,$$

where Y_t is the aggregate production. Therefore, defining $x_{it} \equiv y_{it} - c_{it}$, we can write the market clearing conditions for each consumption good as:

$$c_{it} = y_{it} - x_{it}, \quad i \in \{a, s\}. \tag{2-14}$$

As a result, the law of motion of capital is then given by

$$p_{at}x_{at} + p_{mt}x_{mt} + p_{st}x_{st} = K_{t+1} - (1-\delta)K_t. \tag{2-15}$$

Definition (Competitive Equilibrium). *A competitive equilibrium consists of a sequence of prices $\{R_t, p_{at}, p_{mt}, p_{st}\}$ and feasible allocations for the firms $\{k_{at}, n_{at}, k_{,t}, n_{mt}, k_{st}, n_{st}\}$ and the household $\{c_{at}, c_{mt}, c_{st}, K_{t+1}\}$, such that*

(i) *The sequence $\{c_{at}, c_{mt}, c_{st}, K_{t+1}\}$, given prices, solves the household's problem;*

(ii) *Given prices, the sequences $\{k_{it}, n_{it}\}$, $i \in \{a, m, s\}$, solve firms problems each period;*

(iii) *Markets clear: equations* (2-10), (2-11), (2-14) *and* (2-15) *hold.*

Normalizing the price of the manufacturing sector to one, the model implies that the price of good i relative to manufacturing is a function of the ratio of labor productivity in these sectors:

$$p_{it} = \left(\frac{A_{mt}}{A_{it}}\right)^{1-\alpha}, \quad i \in \{a, s\}. \tag{2-16}$$

[3]This result comes from

$$\frac{k_{at}}{n_{at}} = \frac{k_{at}}{n_{at}}n_{at} + \frac{k_{at}}{n_{at}}n_{mt} + \frac{k_{at}}{n_{at}}n_{st} = \frac{k_{at}}{n_{at}}n_{at} + \frac{k_{mt}}{n_{mt}}n_{mt} + \frac{k_{st}}{n_{st}}n_{st} = k_{at} + k_{mt} + k_{st} = K_t.$$

Production therefore can be aggregated as:

$$Y_t = p_{at}y_{at} + y_{mt} + p_{st}y_{st} = K_t^{\alpha}A_{mt}^{1-\alpha}.$$

For the three sectors, the exogenous components of our model are: sectoral productivity $\{A_{at}, A_{mt}, A_{st}\}$ and investment shares $\{\mathcal{X}_{at}, \mathcal{X}_{mt}, \mathcal{X}_{st}\}$. The equilibrium of the economy can be characterized by a set of 8 equations:

- Given current values of aggregate consumption, capital and A_{mt}, the Euler equation and the law of motion of capital pin down consumption and capital in the next period;
- Three equations determine sectoral consumption, given sectoral productivity and aggregate consumption;
- Finally, three equations pin down sectoral labor share given sectoral consumption and investment shares.

Due to sectoral productivity growth, these eight equations are non-stationary. To deal with this issue, we follow the approach of Hayashi and Prescott (2008): we detrend each variable by manufacturing productivity and rewrite the equilibrium equations using the detrended variables. The use of detrending gives rise to an important feature, which is the fact that the non-homothetic parameters $\bar{a}$ and $\bar{s}$ decrease in importance as manufacturing productivity grows. This means that there is a time horizon long enough in which these parameters are no longer relevant for the equilibrium of the economy. As in Hayashi and Prescott, assuming constant productivity growth in the long run, the system therefore has a steady state with homothetic preferences, in which sectoral labor shares are affected by parameters a, b and ρ. Appendix B gives more details about the system of equations.

3
Calibration

In order to test if our model is able to generate structural transformation as seen in data, we start by using the U.S. economy as a benchmark. This is the first step for eventually analyzing how sectoral productivity evolved across countries. Since capital stock is generated endogenously in our model, in order to solve it, we need sectoral productivity and sectoral investment shares as exogenous inputs. In order to calculate sectoral productivity, we use data on sectoral real value added and hours worked from 2007's version of the 10-Sector Database, by Groningen Growth and Development Centre (GGDC). The series of capital stock is obtained from the Penn World Table (PWT) to create the series of sectoral labor productivity. We then combine compute sectoral productivity using equation (2-13).

We treat investment shares as exogenous, but time-varying. For the United States, data on sectoral investment is not available in national accounts. However, Herrendorf et al. (2013) develop a methodology that splits sectoral value added between consumption and investment. From the data provided by the Bureau of Economic Analysis (BEA), they combine the value added data from the income side of the National Income and Production Accounts (NIPA) with the final expenditure side of the NIPA. The authors then are able do determine the investment and consumption shares of sectoral value added for the U.S. from 1947 to 2010.

In order to abstract from short-run fluctuations, we extract the trend of all series using the Hodrick-Prescott filter with a smoothing parameter $\lambda = 100$. The data on investment shares provide a good explanation to why it is important not to simplify the model assuming that manufacturing is the only sector responsible for capital accumulation. As Figure 3.1 shows, the manufacturing's share of investment in 1950 was roughly 68%, while services' share was around 25%. In 2005, however, investment share of manufacturing decreased to a level close to 52%, while investment from services rose to almost 48%. The share of investment from agriculture decreased from roughly 2% in 1950 to less than 1% in 2005.

As usual, we have to choose parameters values for a, b, $\bar{a}$, $\bar{s}$, α, β, δ, ρ, and the time series of labor productivity, A_{it} and investment shares $\mathcal{X}_{it}$, for the entire time series and $i \in \{a, m, s\}$. First, we calibrate the capital share, α, as 0.33. This value is in line with other works that study capital shares across countries, such as Gollin (2002). The values of β and δ are set as 0.96 and 0.1,

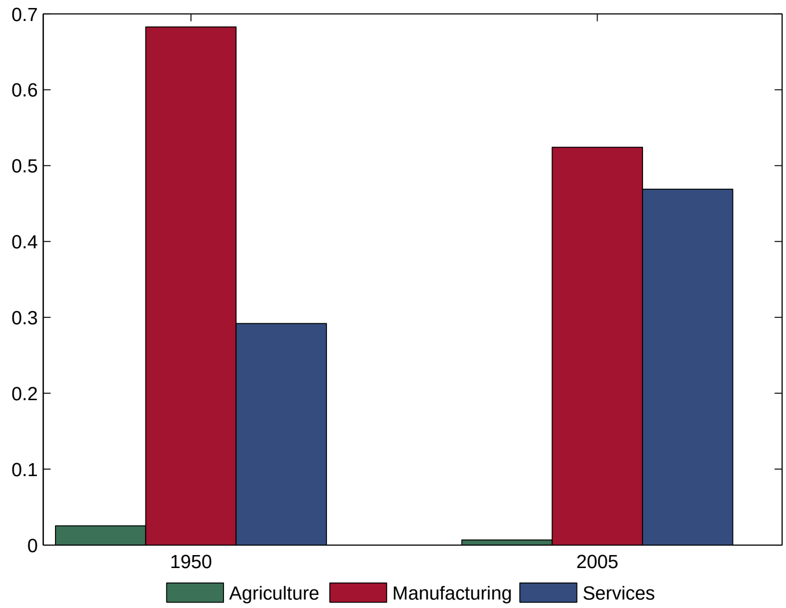

Figure 3.1: Share of Aggregate Investment by Sector

respectively. These values are also in line with structural transformation and growth literature. We follow the literature on structural transformation (see, for example, Duarte and Restuccia (2010)) and set the parameter a equal to 0.1.

To pin down the values of b and ρ, we use equations (7-9)–(7-14), since they affect the steady state of the model. Using the calibrated parameters described above and setting the steady state growth rate of manufacturing sector as the average growth rate of the last ten years of our sample, we then choose the values of b and ρ such that steady-state employment of manufacturing is close to 15% and employment in service sector is close to 85%. According to Figure 1.1, these values seem reasonable, since countries with very high GDP per capita have employment shares close to these values.

Finally, parameters $\bar{a}$ and $\bar{s}$ are chosen so that employment shares in agriculture and manufacturing are equal to the actual shares found in the data for the first period of the sample. These are the only parameters in our calibration that are deliberately chosen to match the U.S. data. Table 3.1 summarizes the calibrated parameters.

Our model solution is based on Hayashi and Prescott (2008). After the steady state is solved, we implement a *shooting algorithm* for the capital stock and aggregate consumption, taking $\tilde{\lambda}_0^{-1}$ as the jumping variable. The algorithm starts by detrending the capital stock for the first period of our sample to get $\tilde{K}_0$ and sets a time horizon of T such that the system of equations (7-1) and (7-2) is sufficiently near the steady state. The algorithm then adjusts $\tilde{\lambda}_0^{-1}$ up if

Table 3.1: Parameter Values

Parameter	Value	Target
$\{A_{it}\}_{t=1950}^{2005}$	$\{\cdot\}$	Actual productivity growth
α	0.33	Literature
β	0.96	Literature
δ	0.1	Literature
a	0.01	Literature
b	0.005	$n_m \approx 15\%$ in steady state
ρ	-0.8	$n_s \approx 85\%$ in steady state
$\bar{a}$	0.4679	Share of hours in agriculture in 1950
$\bar{s}$	9.4112	Share of hours in manufacturing in 1950
$\{\mathcal{X}_{it}\}_{t=1950}^{2005}$	$\{\cdot\}$	Herrendorf et al. (2013)

$(\tilde{K}_T, \tilde{\lambda}_T^{-1})$ is below the steady state; if $(\tilde{K}_T, \tilde{\lambda}_T^{-1})$ is above the steady state, it adjusts $\tilde{\lambda}_0^{-1}$ down. After the algorithm surpasses the time period of our sample (56 years), we set parameters $\bar{a}$ and $\bar{s}$ equal to zero to guarantee that the model converges to its steady state with homothetic preferences. After the sequence of $\{\tilde{K}_t, \tilde{\lambda}_t^{-1}\}$ is obtained, we use equilibrium equations (7-3)–(7-8) to pin down the path of the share of hours worked in the three sectors.

Figure 3.2 presents the path of labor shares predicted by the model when compared to actual data. Since, in the data, productivity grows on average 4.1%, 1.2% and 0.6% in agriculture, manufacturing and services, respectively, higher productivity growth in agriculture move labor away from this sector. Likewise, higher productivity growth in manufacturing compared to services also drives labor away from the first sector to the second. Income effects due to increasing productivity and capital stock reinforces this process. Hence, both mechanics described in Kongsamut et al. (2001) and Ngai and Pissarides (2007) are present here. Although the dynamics of labor share in manufacturing is not exactly hump-shaped, the labor shares at the end of the period are almost identical to the data. Note that the employment shares predicted by the model match almost perfectly the end-of-sample shares from the data even though the parameters were not calibrated to match this feature, only the long-run trend assumed by the cross-country data.

The model is also able to predict the dynamics of sectoral value added. For example, the model is able to predict the downturn of value added per hour worked of agriculture, although in a different time period from the data. However, the model also predicts a sharper downturn in value added per hour worked in manufacturing in the 1980's. The dynamics predicted by services value added, however, broadly predicts the data. Figure 3.3 displays these

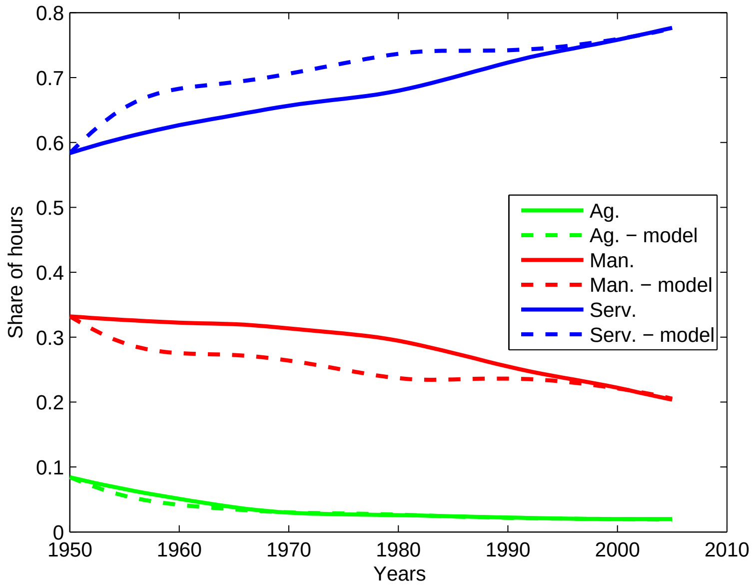

Figure 3.2: Share of Hours by Sector - Model vs. U.S. Data

results. The reader can also see that the capital stock predicted by the model is very similar to the path observed in data.

Using equation (2-16), we can also use sectoral productivity to predict average growth in relative prices. From data, we compute implicit producer price deflators for each sector using data on sectoral value added at constant and current prices from the GGDC database. In data, the price of agricultural goods relative to manufacturing decreased 1.7% on average, while the model predicts an decrease of 2.2%. Additionally, the price of services relative to manufacturing increased 0.8% on average, while in the model it increases 0.2%. That is, the model broadly predicts all the important features of sectoral activity of the U.S. economy.

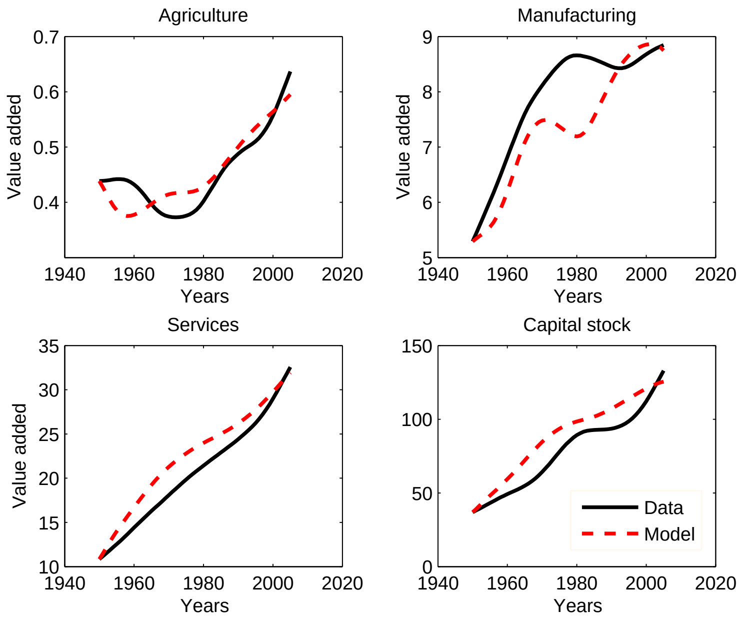

Figure 3.3: Value Added by Sector and Capital Stock–Model vs. U.S. Data
Values in Millions of 2005 U.S. dollars per hour worked.

4
Cross-Country Analysis

We now extend our analysis to assess the quantitative effects of sectoral labor productivity, investment and capital stock on structural transformation across the panel of twenty six countries present in the GGDC 10-Sector database. However, due to very small values of $\tilde{K}_0$ for eight countries, the shooting algorithm breaks down due to its inability to choose $\tilde{\lambda}_0^{-1}$ such that the dynamical system is on the stable saddle path converging to the steady state. Therefore, we keep only eighteen countries in our database, United States included.

We leave preference parameters unaltered, as in the benchmark economy, adjusting only the non-homothetic parameters for each country to match initial labor shares and proceed in three steps. We set initial sectoral productivity for the United States equal to one in 1950, in order to assess how sectoral productivity of other countries evolved over time. We then use the model to generate structural transformation and analyze the model predictions about sectoral employment and aggregate productivity. Finally, we perform some counterfactual exercises to understand the importance of sectoral productivity, investment and capital stock to explain structural transformation across countries.

4.1
Sectoral Productivity Across Countries

We start this subsection explaining how we merge the GGDC database with PWT. Since real value added per sector in GGDC database is denominated in local currency and aggregate capital stock in PWT is denominated in U.S. dollars in Purchase Power Parity in 2005, we proceed in the following way: in the PWT database, we calculate the capital-output ratio on the year in which the real value added is indexed in the GGDC database. We then use this value to convert capital to the local currency, and use the behavior of the PWT series to generate a series of capital stock in local currency at constant prices. Although this procedure is not entirely precise, since capital stock is in PPP values, this is the only way we can merge databases that are reliably comparable across countries. This enables us to calculate sectoral productivity for all countries as in equation (2-13).

We now use the model to restrict the level of sectoral productivity for each sector in the first period. As argued in Duarte and Restuccia (2010), this step is needed because of the lack of PPP-adjusted sectoral output data across a large set of countries. What we do is use the ratio of capital stock and hour worked from each country to compute the its value relative to the United States in the first period of the sample. Next, we use the initial productivity derived from Duarte and Restuccia's framework to pin down our sectoral productivity relative to U.S..[1] Their approach is to choose sectoral productivity so as to match three targets from the first year of the sample: i) aggregate productivity relative to U.S.; ii) share of hours in agriculture; iii) share of hours in manufacturing. Since they use a model without capital, we calculate sectoral productivity for sector i in country j from $Prod_{ij} = A_{ij}^{1-\alpha} K^{\alpha}$, where $Prod_{ij}$ is the initial productivity calculated using Duarte and Restuccia's framework in out database and K is the value of capital-hours worked ratio relative to the United States.

Figure 4.1 plots the average level of sectoral productivity and capital relative to the level of the United States for countries in each sextile of aggregate productivity in the first year. As expected, the model implies that sectoral productivity tend to be lower in poorer countries. In agriculture, sectoral productivity, up to the fourth sextile, is less than 23% of the U.S. productivity. Fifth and sixth quintiles, on the other hand, are much more productive in agriculture than the United States. Fifth sextile of aggregate productivity has a mean agricultural productivity of 5.9, while the sixth sextile of aggregate productivity has a mean agricultural productivity of 4.5. The reader can also see that sectoral productivity is very low relative to the United States in the first two sextiles in manufacturing and services. In manufacturing, the fourth sextile has an average productivity almost equal to the United States. In services, the same happens in the fifth sextile. The reason why some sextiles display high relative productivity levels is because relative capital stock by hour worked is at most 40.8% for the fifth sextile, and on average 18.6%. Since capital stock is low, relative productivity is higher in out framework than in the one from Duarte and Restuccia.

We then use equation (2-13) to calculate sectoral productivity across time through countries and these growth rates to analyze how sectoral productivity evolved for each country until the end of our sample. Figure 4.2 plots the average sectoral productivity in our sample in first and last years, divided by sextiles of countries. It shows that, despite sectoral productivity growth in these countries, productivity growth in the United States was so strong that relative productivity in these countries actually decreased. This phenomenon is more pronounced in the fifth quintile of agriculture: relative productivity decreases

[1]The authors set sectoral productivity for the United States equal to one for each sector.

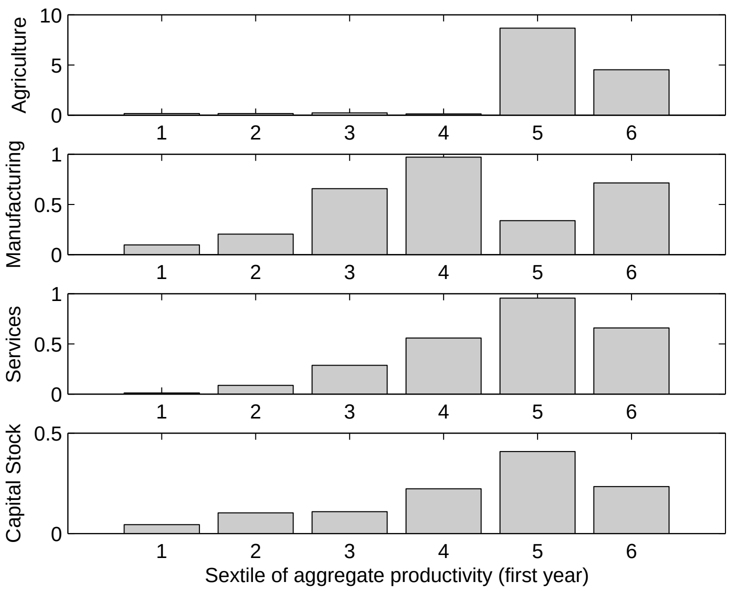

Figure 4.1: Relative Labor Productivity and Capital Stock across Sectors–First Year

from more than eight times U.S. productivity to a little more that twice its north-american counterpart. Relative productivity experiences increase in the fourth sextile of agriculture (from 13% to 32%), in the first, second, fifth and sixth sextiles of manufacturing (from 10% to 11%, from 20.5% to 20.9%, from 34% to 45% and from 72% to 74%, respectively). In services, apart from a negligible increase in relative productivity in the first sextile, only the sixth presents increase–from 66% to 84%.

Next, we compare our results with Duarte and Restuccia's framework applied to our database. Figure 4.3 plots sectoral productivity in the first year for both frameworks. The reader can note that the most important effect of considering capital in our model is on agricultural productivity: the level of the average of the fifth and sixth sextiles of countries in our sample are both around the triple of the ones found in Duarte and Restuccia's model without capital. In manufacturing and services, on the other hand, this effect is more diffuse. These results suggest that poor and highly developed countries have both low capital stock and sectoral productivity relative to the United States, while the countries between the second and fifth sextiles usually compensate their relatively low capital stock with a large relative sectoral productivity.

Figure 4.4 plots sectoral productivity in the last year for both frameworks. Relative productivity in agriculture and manufacturing are on average higher in the framework with capital, but in agriculture the framework without capital has higher level of productivity except for the fifth sextile. In services, productivity is on average higher in the framework without capital, but in the

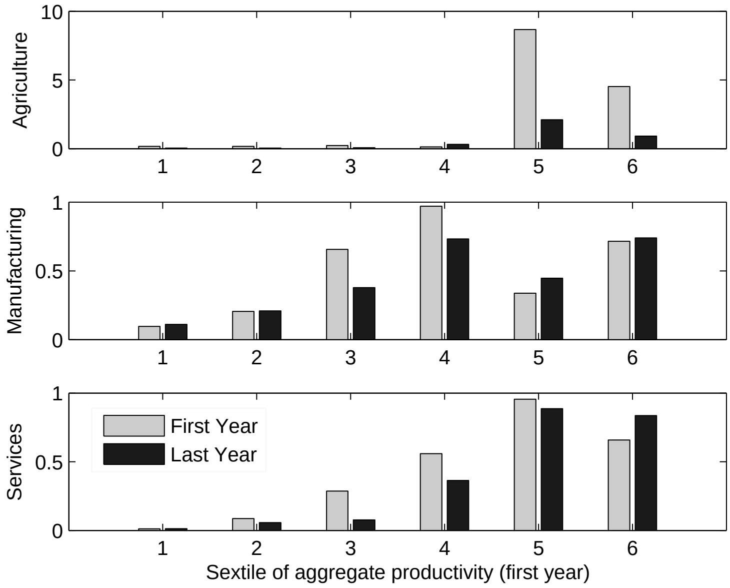

Figure 4.2: Relative Labor Productivity across Sectors–Last Year

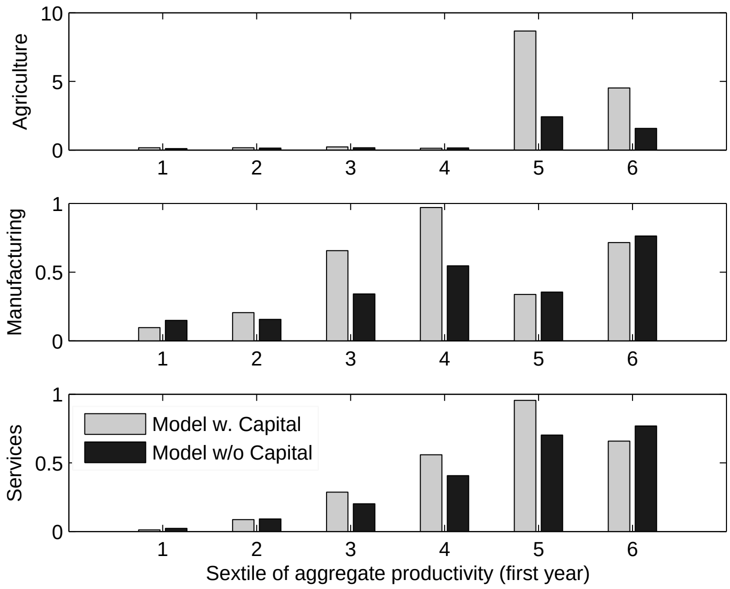

Figure 4.3: Relative Labor Productivity across Sectors–First Year
Relative productivity from both the model with and without capital.

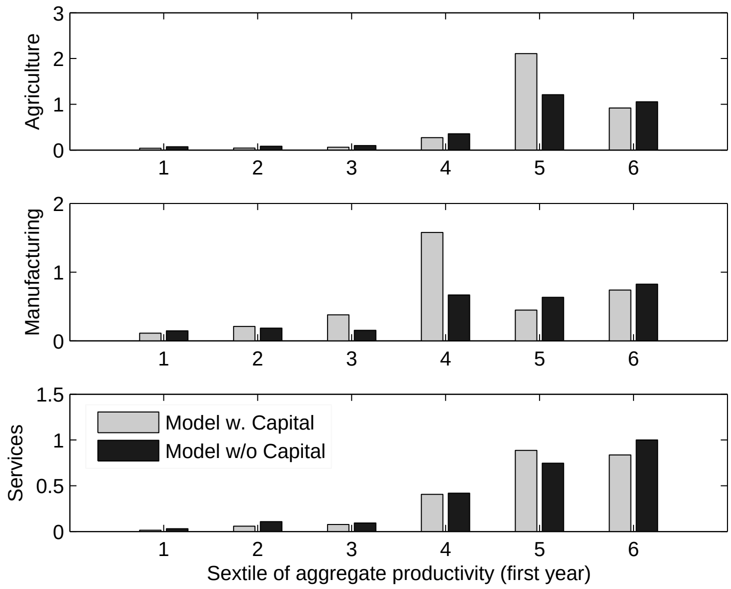

Figure 4.4: Relative Labor Productivity across Sectors–Last Year
Relative productivity from both the model with and without capital.

fifth sextile the framework with capital has higher level of labor productivity.

4.2 Generating Structural Transformation in Other Countries

Next, we use our model to solve for the dynamics of labor share for each country in our sample. We start each country from their first-period detrended capital and use equations (7-1) and (7-2) to solve for the path of capital stock. We use investment share from the United States for this exercise, since this is the only reliable source we have for an extended time period.

The model broadly explains the dynamics of sectoral labor within each country. Figure 4.5 plots the end of the period labor share for each country. The model performs particularly well in explaining the path of agricultural labor. In manufacturing and services, the model neither overpredicts nor underpredicts the end of the period labor share, in general. However, for some of the countries analyzed, our model predicts a relatively larger share of labor in manufacturing than seen in data. With respect to relative aggregate productivity, the model also fits the 45^0 line reasonably well, suggesting that it can broadly match the end-of-sample relative productivity derived from the data.[2] The correlation coefficient between the end-of-sample values for the labor share in agriculture, manufacturing and services is 0.93, 0.01 and

[2]For country i and sectors $j \in \{a, m, s\}$, we use $\frac{\sum_j A_{jt,i}^{1-\alpha} K_{t,i}^{\alpha}}{\sum_j A_{jt,US}^{1-\alpha} K_{t,US}^{\alpha}}$ as the measure of relative aggregate productivity both from the model and from the data.

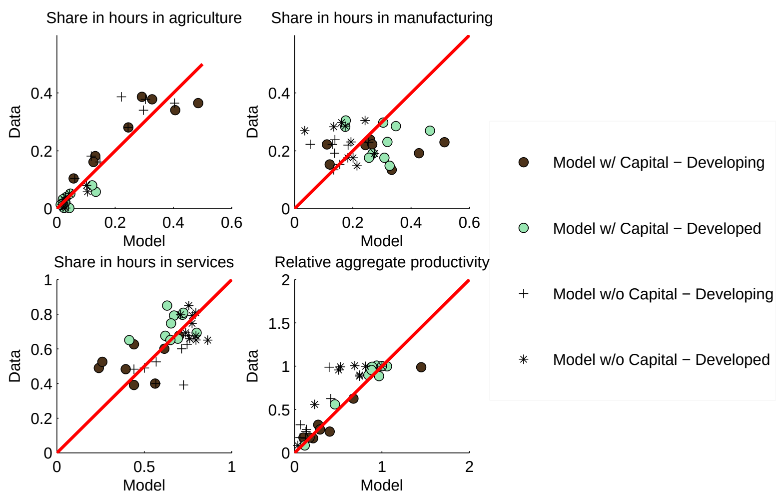

Figure 4.5: Model vs. Data across Countries–Levels in the Last Year

Each plot reports the value for each variable in the last period for the model and the data.

0.66, respectively, suggesting a rather poor performance in manufacturing. With respect to relative productivity, the correlation coefficient is 0.97. When dividing our sample into developing and developed countries,[3] the sectoral labor share correlation in the end of sample is 0.89, 0.11 and 0.39 for developing countries and 0.78, -0.11 and 0.31 for developed countries. With respect to relative aggregate productivity, correlation is 0.98 and 0.99, respectively.

In general, the performance of the model is better in developed countries, especially in the agricultural sector. To verify this statement correctly, we calculated the mean absolute deviation for the entire path of each labor share for both groups of countries. As Table 4.1 shows, the mean absolute deviation in the group of developed countries is approximately 2 p.p. and 8.5 p.p. for emerging countries for agriculture. In manufacturing, the mean absolute deviation is approximately 7.6 p.p. for developed countries and 6 p.p. for emerging economies. Finally, The mean absolute deviation in the service sector is 7.9 p.p. for developed countries and 11 p.p. for emerging ones.

How does the model performance compare to the framework without capital developed by Duarte and Restuccia? We use their framework in our data to compare the two frameworks. Figure 4.5 also plots sectoral labor shares predicted by their framework in the last period of our sample. Both models seem to perform reasonably close overall. One thing to notice is that the model without capital tends to underpredict the service employment share, while our framework tends to overpredict it. The opposite happens in the manufacturing sector. The correlation coefficient between the data and the share of labor by

[3]See Appendix B for a complete list of the countries considered in this section.

Table 4.1: Mean Absolute Deviation

Model	Agriculture	Manufacturing	Services
With Capital	4.8796	6.9108	9.2723
Without Capital	3.7663	6.6608	9.215
With Capital - Emerging	8.4604	5.995	11.033
Without Capital - Emerging	6.3289	4.4879	9.7676
With Capital - Developed	2.0149	7.6434	7.8639
Without Capital - Developed	1.7162	8.3992	8.773

Mean absolute deviation is measured in percentage points between the time series in the models and the data across countries.

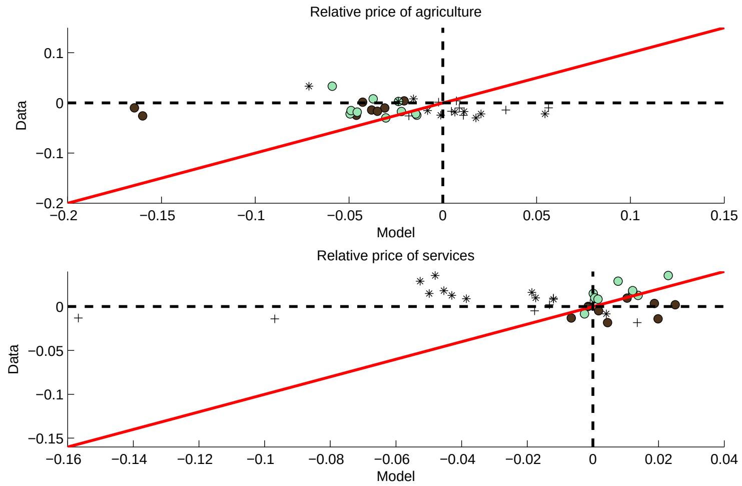

Figure 4.6: Changes in Relative Prices

Each figure reports the annualized change of the variable in the time series in the data and in the model. Relative prices of agriculture and services refer to the prices of agriculture and services relative to manufacturing.

sector predicted by their model is equal to 0.94, -0.11 and 0.64 for agriculture, manufacturing and services, respectively. With respect to relative aggregate productivity, the correlation equals 0.92.[4] When using the whole time series of employment shares to compare the two frameworks, Table 4.1 shows that both models perform similarly well. Here, we use the mean absolute deviation of each time series to assess the goodness of fit of each model. The Table shows that the bulk of the slightly better aggregate performance of Duarte and Restuccia's framework without capital is explained by the worst performance of our model in developing countries. When considering only developed countries, allowing for endogenous capital accumulation improves the predictions of employment paths.

[4]For Duarte and Restuccia's framework, we use $\frac{\sum_j Prod_{jt,i}}{\sum_j Prod_{jt,US}}$ as the measure of relative aggregate productivity.

What could explain our model's worse performance in developing countries, especially for some countries in Latin America, such as Bolivia and Brazil, and also South Korea? Some forces can be in play here. The first one, suggested by Duarte and Restuccia, can help explain the loss of prediction power for both models. For less developed countries, frictions in labor reallocation may be important in accounting for their structural transformation.[5] For our model, the hypothesis of perfect capital mobility between sectors and homogeneous capital can also explain these less satisfying results. Finally, one could argue that data on sectoral output and capital stock is less reliable for developing countries, so that sectoral productivity calculated as a residual of equation (2-13) is too noisy or full of measurement errors.

Using 10-Sector database, we can also compute the dynamics of relative prices for all countries in our sample. Figure 4.6 plots the average price growth of agriculture and services relative to manufacturing. We also compute in Figure 4.6 the predicted average relative prices of agriculture and services using Duarte and Restuccia's framework.[6] Both models badly predict the overall change in relative prices in agriculture, but our framework performs a little better. The correlation between relative price changes in agriculture implied by the data and the one implied by our model is 0.06, while the correlation between the average change predicted in the model without capital is -0.62. On the other hand, our model broadly predicts the implied average change in relative prices in data. The correlation between the predicted and actual average change is 0.32. Duarte and Restuccia's framework, however, still performs badly, with a correlation coefficient of 0.05. In both models, only growth in sectoral productivity drive relative price changes over time. Other factors affecting such price changes over time are not captured in these frameworks.

4.3 Capital and Productivity as Drivers of Structural Transformation

Duarte and Restuccia, in their work, assessed the effects of sectoral productivity growth in explaining structural transformation and aggregate productivity. Here, we reevaluate their work through the lens of our model, in which sectoral output per hour worked is now split between capital stock and sectoral productivity. This step is important to understand how the each of

[5]Some studies, such as Duarte and Restuccia (2007), Dekle and Vandenbroucke (2012) and Hayashi and Prescott (2008) introduce labor market frictions to account for country-specific factors affecting labor reallocation.

[6]Relative prices dynamics in our model are calculated using equation (2-16). In Duarte and Restuccia (2010), sectoral prices relative to manufacturing are computed using $p_{it} = \frac{Prod_{mt}}{Prod_{it}}$.

these effects explain structural transformation across countries. To do this, we first analyze the effect of sectoral productivity growth in the process structural transformation in the context of both models. Then, we assess the effect of no capital accumulation using Duarte and Restuccia's model and discuss the role of capital in explaining sectoral labor reallocation.

Since our model performs unsatisfactorily in developing countries in the baseline calibration, we choose to withdraw them from these exercises, since counterfactual analysis would not render fruitful results. Using only developed countries, we also had to drop Hong Kong and Italy from our sample due to the fact that the shooting algorithm was unable to choose $\tilde{\lambda}_0^{-1}$ such that the dynamical system is on the stable saddle path converging to the steady state for most of the exercises.

4.3.1 Sectoral Productivity and Structural Transformation

To assess the quantitative effect of sectoral productivity growth in the process of structural transformation, we start by setting growth rates equal to zero. Since our model features capital accumulation, even though these growth rates might be zero, sectoral labor reallocation can still occur.

We begin by setting all growth rates equal to zero. Much of the labor force is kept in agriculture relative to the baseline prediction of the model. As a consequence, most of the labor force is not reallocated to manufacturing and services. The same effect takes place when we set only agricultural productivity growth equal to zero. When compared to the model from Duarte and Restuccia, in their setting, an even greater proportion of labor stays in agriculture relative to our model. In addition, in their framework, this in turn penalizes the services sector more than manufacturing. Setting productivity growth in services equal to zero has no effect on drawing labor away from agriculture. The effect on manufacturing and services is small, but mostly contrary in both frameworks: in Duarte and Restuccia's, more labor force is drawn away from manufacturing and directing to services, while in our the opposite occurs.

The most important result appears when we set manufacturing productivity growth equal to zero: in Duarte and Restuccia's framework, there is much less reallocation of labor away from industry into services compared to the original prediction of the model. However, in our model, this does not affect substantially our initial results. The cause of this difference is mostly due to the measures of productivity in each model, which we will discuss shortly. Table 4.7 presents the key results from our discussion.[7]

[7]We leave the complete results of this section to Appendix 7.

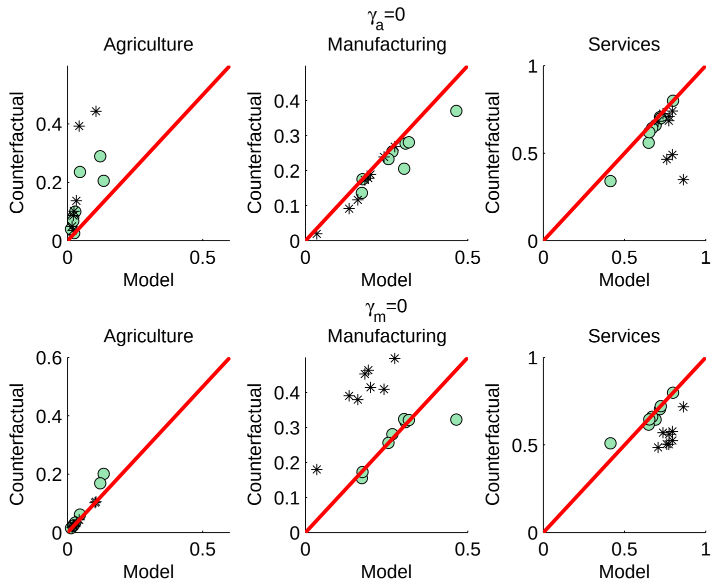

Figure 4.7: The Effect of Zero Productivity Growth

Line one sets the growth rate of agricultural productivity equal to zero in all countries, leaving the other sectors as in the data and two set, while the second line does the same for manufacturing. Each line plots the sectoral labor share in the end of the sample for the baseline calibration and the counterfactual exercise.

The results from our model suggest that productivity growth in agriculture is indeed an important factor driving labor away from agriculture and manufacturing and directing them to services, in line with Gollin et al. (2002) found in their study. This, however, has little effect on aggregate relative productivity in all cases analyzed, which is largely due to the fact that the United States also has no productivity growth in all sectors.

Following Duarte and Restuccia and using sectoral productivity growth derived from U.S. data, the results regarding structural transformation are barely affected in both models. The most important results are related to aggregate productivity (see Figure 4.8). In our model, it increases substantially when we set the growth rates in manufacturing productivity equal to the United States: more than ten times the baseline model. We believe part of this effect is explained by the dynamic equations from our model, in which the growth rate in the manufacturing sector affects the dynamics of aggregate capital. Aggregate productivity increases substantially in their framework when productivity growth in services grows at the same rate as the United States for all countries: almost five times the baseline model, on average, which does not happen in our model. This might be explained by the fact that their measure of productivity embeds both our sectoral productivity and capital stock as well. This might be forcing a stronger capital accumulation in this sector than would be feasible in a general equilibrium model with actual

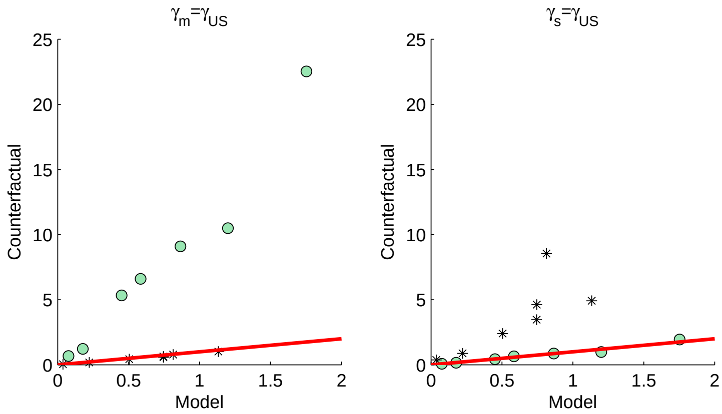

Figure 4.8: Aggregate Productivity When Setting Productivity Growth Equal to the United States

capital stock.

4.3.2 The Role of Capital Accumulation

We now attempt to understand how capital stock affects the process of structural transformation. In order to understand how this factor of production might affect labor reallocation, we use the same model developed in Duarte and Restuccia (2010) and consider their measure of labor productivity as the combination of sectoral productivity and capital stock. Here, however, we will consider capital stock as completely exogenous, which implies that no intertemporal decision is made by the representative household. Thus, equilibrium labor shares in this framework are described by the following two equations:[8]

$$n_{at} = (1-a)\frac{\bar{a}}{K_t^{\alpha}A_{at}^{1-\alpha}} + a\left(1 + \frac{\bar{s}}{K_t^{\alpha}A_{st}^{1-\alpha}}\right) \tag{4-1}$$

$$n_{mt} = \frac{(1-n_{at}) + \frac{\bar{s}}{K_t^{\alpha}A_{st}^{1-\alpha}}}{1+x_t}, \tag{4-2}$$

where

$$\left(\frac{b}{1-b}\right)^{\frac{1}{\rho-1}}\left(\frac{K_t^{\alpha}A_{mt}^{1-\alpha}}{K_t^{\alpha}A_{st}^{1-\alpha}}\right)^{\frac{\rho}{\rho-1}} \equiv x_t.$$

Figure 4.9 plots the results of changing only the exogenous path of sectoral productivity A_{it} while holding capital in the same level as the first period of the sample, relative to the United States. Since this measure of sectoral productivity grows at a different rate than Duarte and Restuccia's measure of labor productivity, changes in sectoral composition of labor will differ from the baseline calibration used to compare our model and their

[8]See Duarte and Restuccia (2010) for further reference.

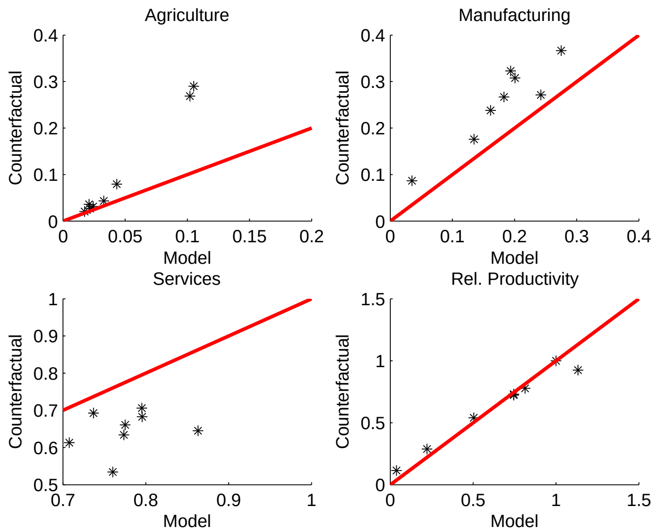

Figure 4.9: The Assumption of Constant Capital

Counterfactual assumes that actual capital stock is constant in the first period of the sample. Each column plots the sectoral labor share and relative aggregate productivity in the end of the sample for the baseline calibration and the counterfactual exercise.

framework. In this setting, extracting the capital accumulation component from the model reduces the rate in which sectoral productivity grows in agriculture. As a result, much less labor is drawn from this sector.

The fact that capital stock does not grow in this setting also affects the results for manufacturing and services. Neglecting capital growth effects in the measure of labor productivity of Duarte and Rescuccia's framework also reduces the growth rate in which sectoral productivity grows. This has the effect of also reducing the amount of labor reallocated to services. As the reader can see in Figure 4.9, the end result of this experiment is a much larger share of labor allocated to agriculture and manufacturing than on the benchmark calibration. As a result, 70% is the upper bound the equilibrium labor share of the services sector in this exercise.

What is interesting is that neglecting capital accumulation effects in the exercise does not affect substantially the levels of countries' aggregate productivities relative to the United States in this exercise. This suggests that capital accumulation, although important for analyzing sectoral reallocation of labor, is much less relevant for the analysis of the aggregate economic performance of countries. This is actually in line with Restuccia (2013), that argues that, when comparing Latin American countries to the United States, differences in the capital-to-output ratio and capital accumulation are not systematic and quantitatively substantial enough to explain differences in GDP per hour between them.

These results suggest that a framework without capital, such as Duarte and Restuccia's, when considering only value added per hour worked as a measure of sectoral productivity, is likely to be misinterpreting the underlying mechanism behind structural transformation. In their framework, what drives sectoral labor reallocation is merely the behavior of a measure of labor productivity that encompasses any factor apart from labor hours employed in the sector. This means that any source of increase in sectoral output that is not due to additional hours worked is embedded in this variable. Hence, when considering the growth in this measure of productivity, their model suggests that its growth in manufacturing sector is the main driver of the second stage of structural transformation, that is, the reallocation of labor from manufacturing to services. Furthermore, in their framework, we have seen that assuming a catch up in productivity growth relative to the U.S. in the service sector implies a significant increase in aggregate productivity. Therefore, one could conclude that productivity growth in manufacturing is supposedly an important ingredient in the process of structural transformation across countries and productivity growth in services is remarkably relevant for an increase in aggregate productivity. These conclusions, however, seem misleading. Since this measure of labor productivity also takes into account capital accumulation, the driving force behind this productivity growth can be largely explained by an increase in capital stock per hour worked, which implies that capital accumulation, not growth in manufacturing productivity, is the mechanism behind this reallocation. This can be easily illustrated when we take the growth rates of the measures of productivity:

$$\frac{\dot{Prod}_{mt}}{Prod_{mt}} = \alpha\frac{\dot{K}_t}{K_t} + (1-\alpha)\frac{\dot{A}_{mt}}{A_{mt}},$$

which means that their measure of labor productivity will grow at a higher rate than ours if an only if capital stock grows at a higher rate than our measure of sectoral productivity.

Our model suggests that this is what is taking place. Our first experiments in this section actually show that setting productivity growth equal to zero or equal to U.S. sectoral rate has minimal effects on labor reallocation from manufacturing to services. Assuming constant capital, as our last counterfactual, also support this view: much less employment is reallocated from manufacturing to services. In addition, productivity growth catch up in services actually plays little role in the increase in aggregate productivity in a richer model. As should be expected, the bulk of these discrepancies are due to the fact that the growth rate of their measure of labor productivity is much greater than ours. Figure 4.10 plots this evidence for manufacturing. At the end of the

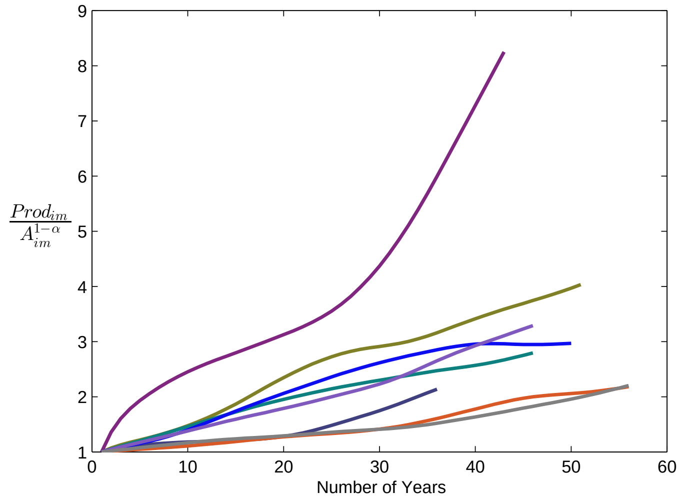

Figure 4.10: The Ratio Between Different Measures of Productivity in Manufacturing

Figure plots the ratio between labor productivity as in Duarte and Restuccia's framework and our measure of sectoral productivity for the countries considered in the counterfactual exercises. For each country, ratios in the first period of the sample are normalized to one. Differences in the length of each line is due to different time series dimensions.

sample, output per hour in manufacturing in the framework without capital is at least the double of manufacturing productivity in the model with capital. In Korea, the first productivity measure is roughly 8.3 times the second. These huge differences are entirely due to capital accumulation.

This does not mean, of course, that the framework employed by recent literature is wrong. However, we emphasize that taking output per hour for granted as the driver of sectoral employment reallocation and aggregate productivity can have adverse effects when one thinks about policy prescriptions. Although none of the frameworks explicitly derive how productivity grows in each sector, it could be argued that policies should be designed in less developed countries to accelerate the growth rate of manufacturing productivity in order to turn the economic structures of such countries and the rich world more alike. It could also be argued that governments should design policies intended to boost productivity in the service sector in order to increase aggregate productivity. Our model suggests that such policy prescriptions are misguided: both of these factors seem to be much more related to capital accumulation than sectoral productivity. Policies intended to boost aggregate productivity should not be sector-specific, but focus on how the economy as whole.

We end this section emphasizing that, although richer models of structural transformation might be subject to obstacles regarding the fit with the data and also its computational implementation, they are valuable to understanding more thoroughly the mechanisms underlying the process of labor reallocation across sectors and its impact on aggregate productivity. Here, adding more structure to Duarte and Restuccia's model pays off by improving our understanding of what is a more proximate cause of the second stage of the process of structural transformation. As far as our framework goes, the answer is much closer to capital accumulation than productivity growth in manufacturing.

4.4
Robustness: The Effects of Different Sectoral Investment Shares

Since the beginning, we have been using the time series of sectoral investment share as calculated by Herrendorf et al. (2013). This assumption can be considered rather strong, since one can argue that the United States' economic structure is different from other countries, which could affect how sectors build capital. To check if our quantitative exercise is robust to variations in the amount each sector is responsible for aggregate investment, we will now allow for different assumption about how investment is made in the economy.

We begin by making the assumption that all investment is made by manufacturing. This is a fairly popular assumption in three-sector models of structural transformation with capital, present in works such as Kongsamut et al. (2001). Therefore, it is important to analyze the effects of such assumption in our quantitative analysis. In our model, this means that we set $\mathcal{X}_{mt} = 1$ for all t, while the others are set to zero. Since agriculture had almost no presence in the amount of investment made since the beginning of our sample, this in effect changes virtually nothing of agricultural labor share. On the other hand, this assumption predicts a much larger labor share in manufacturing and a smaller share in services than in the baseline calibration. Since there was no obvious bias in our baseline calibration, this suggests that making such assumption largely biases the predictions of structural transformation, at least in our model. These results are displayed in the first column of Figure 4.11.

Next, we assume that the investment shares have not changed in our sample since its beginning. The motivation behind this exercise lies on the fact that most countries in our sample are not as developed economically as the United States. Hence, assuming that sectoral investment shares change through time in the same way as in the U.S. can be a rather strong assumption. The result of this exercise, as seen in the second column of Figure 4.11, has the effect of only increasing the share of labor in manufacturing at the end of the sample for all countries, while slightly diminishing the share of labor in services.

Finally, we make a use sectoral investment shares data from a less de-

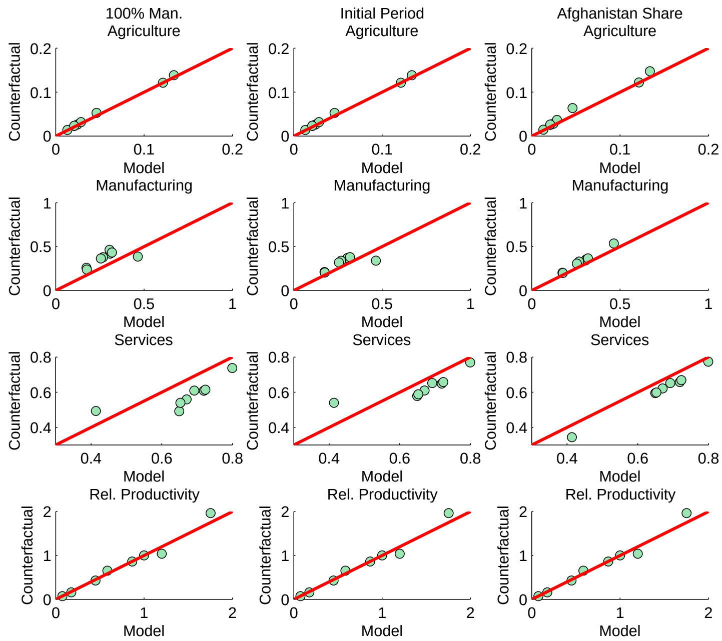

Figure 4.11: Labor Allocation Assuming Different Sectoral Investment Shares

Column one assumes that manufacturing is responsible for all the investment in the economy. Column two keeps the same sectoral investment share as seen in the first period of the sample calculated by Herrendorf et al. (2013). Column three assumes that the investment share is equal to the average in Afghanistan from 2003 to 2010. Each column plots the sectoral labor share and relative aggregate productivity in the end of the sample for the baseline calibration and the counterfactual exercise.

veloped country to analyze the robustness of our previous results assuming the initial shares of the sample. Here, we use data from Afghanistan between 2003 and 2011 due to lack of data availability from other countries. In Afghanistan, the shares of agriculture, manufacturing and services in investment are 6.9%, 60.4% and 30.7%, respectively. As the reader can see, the results are similar to assuming the sectoral investment share from 1950 in the United States. Labor force allocated to agriculture is virtually the same as in our baseline analysis, labor share in manufacturing increases marginally, while the opposite takes place in services. Aggregate productivity, in all of our exercises in this section, is barely affected.

5
Concluding Remarks

In this paper, we have built a three-sector growth model that allows for capital accumulation and also for sectoral investment to be made by all three sectors. This is a novel approach in the literature, since quantitative exercises either assume that only manufacturing can produce capital or simplify the framework to a two-sector model. The model performs well in a panel of countries, broadly explaining the process of structural transformation across countries. The dynamics of relative prices, on the contrary, is not correctly captured by the model, but it still outperforms the framework without capital from Duarte and Restuccia (2010).

This framework is used do assess the results from previous literature, especially Duarte and Restuccia, that emphasize the role os sectoral productivity on structural transformation and aggregate productivity. From counterfactual exercises, we have learned that sectoral productivity growth, especially in agriculture, is important to explain the dynamics of sectoral reallocation across sectors, although it has little effect in explaining the behavior or relative aggregate productivity in the countries analyzed. The same pattern is observed when we use Duarte and Restuccia's framework to investigate the effect of no capital accumulation, albeit its effect is the same as assuming a lower growth rate in sectoral productivity. In contrast to the authors' framework, sectoral productivity growth in manufacturing and services play a much smaller role in explaining sectoral labor reallocation. We also find that productivity growth in services plays a much smaller role in our framework than in theirs with respect to its impact on aggregate productivity. We also found that neglecting capital accumulation effects when measuring sectoral productivity as output per hour worked affects the dynamics of sectoral employment: much of structural transformation is explained solely by capital accumulation.

We believe that a framework without capital can lead to inaccurate conclusions about the effect of productivity growth in structural transformation and aggregate productivity. Since the authors use output per hour as a measure of labor productivity and this in turn embeds capital itself, the analysis can mistake productivity effects of structural transformation with the role of capital accumulation. We believe it is important to separate both effects when thinking about structural transformation and aggregate productivity, since these conclusions affect whether we think that policies should be directed to improving sector-specific productivity or if policymakers should be more concerned about the overall behavior of the economy, such as incentives for private investment, capital markets structure and measures to improve productivity of the economy as a whole.

Finally, we have seen that different assumptions of sectoral investment shares, while having negligible effects on aggregate productivity, can significantly bias the predicted path of employment shares across sectors.

There are some caveats in our proposed framework, however. First, the fact that relative price changes are badly predicted signals that a richer environment must be considered in order to better understand its dynamics. Second, since the model's prediction is worsened when used to analyze developing economies, it is quite possible that our proposed framework is not suited to them. For instance, there can be frictions in labor and capital markets that are more salient in developing countries that are not being considered here. Works such as Duarte and Restuccia (2007) have dealt with this conditions in the case of labor market frictions. Further research can assess the effects of imperfect capital mobility or substitution in explaining the process of structural transformation experienced in such countries.

6
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7
Appendix

Appendix A: Price Index

The price index for the composite consumption good C_t can be derived by minimizing the cost of achieving a given level of aggregate consumption. We will proceed in two steps. The first step consists of minimizing the cost of the composite consumption good considering only the agricultural and non-agricultural good:

$$\min_{c_{at},\, c_{nat}} p_{at}c_{at} + p_{nat}c_{nat} \qquad s.t. \quad (c_{at} - \bar{a})^a (c_{nat})^{1-a} \geq C_t.$$

Let γ_t be the Lagrange multiplier for the minimization problem. First-order conditions imply that

$$c_{at} - \bar{a} = a\frac{\gamma_t}{p_{at}}C_t$$

$$c_{nat} = (1-a)\frac{\gamma_t}{p_{nat}}C_t.$$

From the definition of C_t, we can isolate γ_t to obtain the aggregate price index:

$$P_t \equiv \gamma_t = \left(\frac{p_{at}}{a}\right)^a \left(\frac{p_{nat}}{1-a}\right)^{1-a}.$$

Next, we derive an expression for the non-agricultural price index. The minimization problem now is:

$$\min_{c_{mt},\, c_{st}} p_{mt}c_{mt} + p_{st}c_{st} \qquad s.t. \quad [bc_{mt}^{\rho} + (1-b)(c_{st} + \bar{s})^{\rho}]^{\frac{1}{\rho}} \geq c_{nat}$$

Let ψ_t be the Lagrange multiplier for this minimization problem. First-order conditions imply that

$$c_{mt} = \left(\frac{b\psi_t}{p_{mt}}\right)^{\frac{1}{1-\rho}} c_{nat}$$

$$c_{st} + \bar{s} = \left[\frac{(1-b)\psi_t}{p_{st}}\right]^{\frac{1}{1-\rho}} c_{nat}.$$

From the definition of c_{nat}, we define ψ_t as the price index of non-agricultural goods:

$$p_{nat} \equiv \psi_t = \left[b^{\frac{1}{1-\rho}} (p_{mt})^{\frac{\rho}{\rho-1}} + (1-b)^{\frac{1}{1-\rho}} (p_{st})^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}.$$

Finally, we arrive at the expression for the aggregate price index:

$$P_t \equiv \left(\frac{p_{at}}{a}\right)^a \left\{ \frac{\left[b^{\frac{1}{1-\rho}} (p_{mt})^{\frac{\rho}{\rho-1}} + (1-b)^{\frac{1}{1-\rho}} (p_{st})^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}}{1-a} \right\}^{1-a}.$$

Appendix B: Equilibrium Equations

The equilibrium of the economy can then be characterized by the following equations:

- Euler equation, in which λ_t^{-1} is the aggregate consumption (see Equation (2-4)):

$$\lambda_t = \beta\lambda_{t+1}(1 - \delta + \alpha K_{t+1}^{\alpha-1} A_{mt+1}^{1-\alpha})$$

- Law of motion of aggregate capital:

$$K_{t+1} = (1-\delta)K_t + K_t^{\alpha} A_{mt}^{1-\alpha} - \lambda_t^{-1}$$

- From household's first order conditions, three equations jointly determine sectoral consumption:

$$c_{at} = \frac{a}{\lambda_t}\left(\frac{A_{at}}{A_{mt}}\right)^{1-\alpha} + \bar{a}$$

$$\frac{b}{1-b}\left(\frac{c_{mt}}{c_{st}+\bar{s}}\right)^{\rho-1} = \left(\frac{A_{st}}{A_{mt}}\right)^{1-\alpha}$$

$$\lambda_t^{-1} = \left(\frac{A_{mt}}{A_{at}}\right)^{1-\alpha}(c_{at}-\bar{a}) + c_{mt} + \left(\frac{A_{mt}}{A_{st}}\right)^{1-\alpha}(c_{st}+\bar{s})$$

- Three market-clearing equations that pin down sectoral employment:

$$c_{at} + \mathcal{X}_{at}(K_t^{\alpha} A_{mt}^{1-\alpha} - \lambda_t^{-1} - p_{at}\bar{a} + p_{st}\bar{s}) = K^{\alpha} A_{at}^{1-\alpha} n_{at}$$

$$c_{mt} + \mathcal{X}_{mt}(K_t^{\alpha} A_{mt}^{1-\alpha} - \lambda_t^{-1} - p_{at}\bar{a} + p_{st}\bar{s}) = K^{\alpha} A_{mt}^{1-\alpha} n_{mt}$$

$$c_{st} + \mathcal{X}_{st}(K_t^{\alpha} A_{mt}^{1-\alpha} - \lambda_t^{-1} - p_{at}\bar{a} + p_{st}\bar{s}) = K^{\alpha} A_{st}^{1-\alpha} n_{st},$$

where $\mathcal{X}_{it}$ is the share of investment made by sector $i \in \{a, m, s\}$.

Now, define the following detrended variables:

$$\tilde{K}_t = \frac{K_t}{A_{mt}}, \qquad \tilde{c}_{it} = \frac{c_{it}}{A_{mt}}, \qquad \tilde{\lambda}_t^{-1} = \frac{\lambda_t^{-1}}{A_{mt}}, \qquad \tilde{p}_{it} = \frac{p_{it}}{A_{mt}^{1-\alpha}}.$$

This enables us to rewrite the equilibrium of the economy using detrended conditions:

$$\frac{A_{mt+1}}{A_{mt}}\tilde{\lambda}_{t+1}^{-1} = \beta\tilde{\lambda}_t^{-1}(1-\delta+\alpha\tilde{K}_{t+1}^{\alpha-1}) \tag{7-1}$$

$$\frac{A_{mt+1}}{A_{mt}}\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \tilde{K}_t^{\alpha} - \tilde{\lambda}_t^{-1} \tag{7-2}$$

$$\tilde{c}_{at} = a\tilde{\lambda}_t^{-1}\left(\frac{A_{at}}{A_{mt}}\right)^{1-\alpha} + \frac{\bar{a}}{A_{mt}} \tag{7-3}$$

$$\frac{b}{1-b}\left(\frac{\tilde{c}_{mt}}{\tilde{c}_{st}+\frac{\bar{s}}{A_{mt}}}\right)^{\rho-1} = \frac{\tilde{p}_{st}^{-1}}{A_m^{1-\alpha}} \tag{7-4}$$

$$\tilde{\lambda}_t^{-1} = A_{mt}^{1-\alpha}\tilde{p}_{at}\left(\tilde{c}_{at} - \frac{\bar{a}}{A_{mt}}\right) + \tilde{c}_{mt} + A_{mt}^{1-\alpha}\tilde{p}_{st}\left(\tilde{c}_{st} + \frac{\bar{s}}{A_{mt}}\right) \tag{7-5}$$

$$\tilde{c}_{at} + \mathcal{X}_{at}\left(\tilde{K}^{\alpha} - \tilde{\lambda}_t^{-1} - \tilde{p}_{at}\frac{\bar{a}}{A_{mt}^{\alpha}} + \tilde{p}_{st}\frac{\bar{s}}{A_{mt}^{\alpha}}\right) = \left(\frac{A_{at}}{A_{mt}}\right)^{1-\alpha}\tilde{K}^{\alpha}n_{at} \tag{7-6}$$

$$\tilde{c}_{mt} + \mathcal{X}_{mt}\left(\tilde{K}^{\alpha} - \tilde{\lambda}_t^{-1} - \tilde{p}_{at}\frac{\bar{a}}{A_{mt}^{\alpha}} + \tilde{p}_{st}\frac{\bar{s}}{A_{mt}^{\alpha}}\right) = \tilde{K}^{\alpha}n_{mt} \tag{7-7}$$

$$\tilde{c}_{st} + \mathcal{X}_{st}\left(\tilde{K}^{\alpha} - \tilde{\lambda}_t^{-1} - \tilde{p}_{at}\frac{\bar{a}}{A_{mt}^{\alpha}} + \tilde{p}_{st}\frac{\bar{s}}{A_{mt}^{\alpha}}\right) = \left(\frac{A_{st}}{A_{mt}}\right)^{1-\alpha}\tilde{K}^{\alpha}n_{st}. \tag{7-8}$$

Therefore, equations (7-1)–(7-8) characterize the equilibrium of the economy:

- Equations (7-1) and (7-2) describe the dynamics of the economy. Aggregate capital and consumption are pinned down;
- Given sectoral productivity and aggregate consumption, equations (7-3) to (7-5) determine sectoral consumption;
- Equations (7-6) to (7-8) determine sectoral employment shares, given sectoral consumption, sectoral productivity and sectoral investment shares.

Finally, we want to find the steady state that underlies system (7-1)–(7-8) in terms of these detrended variables. Before we proceed, however, note that the equilibrium condition (7-3) expresses that, as productivity in the manufacturing sector increases, the non-homothetic parameter $\bar{a}$ becomes increasingly irrelevant to total consumption in agriculture. This means that, in a steady state, the Stone Geary properties for agriculture is not important, suggesting that a homothetic utility in agriculture is a reasonable approach. Moreover, equilibrium condition (7-4) states that productivity growth in manufacturing decreases the relevance of the parameter $\bar{s}$ in total consumption of services. These results suggest that our steady state utility function can be approximated as homothetic.[1]

Therefore, we can simplify our equilibrium conditions in steady state into six equations. Let γ_m be the steady state growth rate of labor productivity in manufacturing. Assuming that A_{mt} grows at a smaller rate than A_{at} in steady state so that the share of investment from agriculture sector converges to zero, the equations that express the steady state of the system can be written as:

$$\gamma_m = \beta(1 - \delta + \alpha \tilde{K}_{ss}^{\alpha-1}) \tag{7-9}$$

$$\gamma_m \tilde{K}_{ss} = (1-\delta)\tilde{K}_{ss} + \tilde{K}_{ss}^{\alpha} - \tilde{\lambda}_{ss}^{-1} \tag{7-10}$$

$$a\tilde{\lambda}_{ss}^{-1} = \tilde{K}_{ss}^{\alpha} n_{a_{ss}} \tag{7-11}$$

$$\tilde{c}_{m_{ss}} + \mathcal{X}_{m_{ss}}(\tilde{K}_{ss}^{\alpha} - \tilde{\lambda}_{ss}^{-1}) = \tilde{K}_{ss}^{\alpha} n_{m_{ss}} \tag{7-12}$$

$$\tilde{c}_{s_{ss}} + \mathcal{X}_{s_{ss}}(\tilde{K}_{ss}^{\alpha} - \tilde{\lambda}_{ss}^{-1}) = \frac{1-b}{b}\left(\frac{\tilde{c}_{s_{ss}}}{\tilde{c}_{m_{ss}}}\right)^{\rho-1} \tilde{K}_{ss}^{\alpha}(1 - n_{a_{ss}} - n_{m_{ss}}) \tag{7-13}$$

$$\tilde{\lambda}_{ss}^{-1} = a\tilde{\lambda}_{ss}^{-1} + \tilde{c}_{m_{ss}} + \frac{1-b}{b}\left(\frac{\tilde{c}_{s_{ss}}}{\tilde{c}_{m_{ss}}}\right)^{\rho-1} \tilde{c}_{s_{ss}}, \tag{7-14}$$

so that now we have six equations for six endogenous variables: $(\tilde{K}_{ss}, \tilde{\lambda}_{ss}^{-1}, n_{a_{ss}}, n_{m_{ss}}, \tilde{c}_{m_{ss}}, \tilde{c}_{s_{ss}})$. Note that parameters a, b and ρ affect sectoral employment shares on steady state, which will be important for our calibration of the model.

Appendix C: Data Sources

We build a panel dataset with annual observations of sectoral employment shares, sectoral value added per hour, aggregate GDP per hour and sectoral investment shares for eighteen countries. The countries covered in our data set are, with sample period in parentheses, Bolivia (1950–2003), Brazil (1950–2005), Chile (1951–2005), Costa Rica (1950–2005), Hong Kong (1974–2005), Indonesia (1971-2005), Italy (1951-2005), Japan (1953-2003), Korea (1963-2005), Netherlands (1960-2005), Peru (1960-2005), Philippines (1971-2005), Singapore (1971-2005), Spain (1956-2005), Sweden (1960-2005), Thailand (1960-2005), United Kingdom (1950-2005) and United States (1950-2005). We divide these countries into two subgroups:

1. Developing countries:

[1]As noticed by 9 and 1, Stone-Geary utility functions are asymptotically homothetic.

- Bolivia
- Brazil
- Chile
- Costa Rica
- Indonesia
- Peru
- Philippines
- Thailand

2. Developed countries:

- Hong Kong
- Italy
- Japan
- Korea
- Netherlands
- Singapore
- Spain
- Sweden
- United Kingdom
- United States

All series are trended using the Hodrick–Prescott filter with a smoothing parameter $\lambda = 100$ before any ratios are computed.

Aggregate Data

We use data on PPP-adjusted real GDP per capita in constant prices ($RGDPL$), population (POP) and capital stock at constant PPP-adjusted 2005 U.S. dollars from Penn World Tables version 8.0. We obtain data on total annual hours actually worked ($HOURS$) from the Groningen Growth and Development Centre (GGDC) 10-Sector Database. With these data we construct annual time series of PPP-adjusted GDP per hour in constant prices for each country as $YLh = RGDPL \times POP/HOURS$.

Sectoral Data

We obtain annual data on hours worked and constant domestic-price value added for agriculture, industry, and services for the countries listed above from the GGDC 10-Sector Database. We define agriculture as Agriculture, hunting, forestry and fishing. Sectors Mining and quarrying; Manufacturing; Electricity, gas and water supply and Construction are labeled under manufacturing. Services comprise Wholesale and retail trade, hotels and restaurants; Transport, storage, and communication; Finance, insurance, real estate and business services; Government services and Community, social and personal services. We compute implicit producer price deflators for each sector using data on sectoral value added at constant and current prices.

Sectoral share of investment in the United States comes from 11. The database is available on Valentinyi's website[2]. For specific details regarding the construction of this data, please see the web appendix of the referred article.

Appendix D: Counterfactuals: Figures

The next two figures show the results for employment shares and aggregate productivity at the end of the sample for developed countries for all exercises done in Section 4.3.1, since some were omitted for the sake of clarity of exposition.

[2] `https://sites.google.com/site/valentinyiakos/Home/papers/st-preferences`

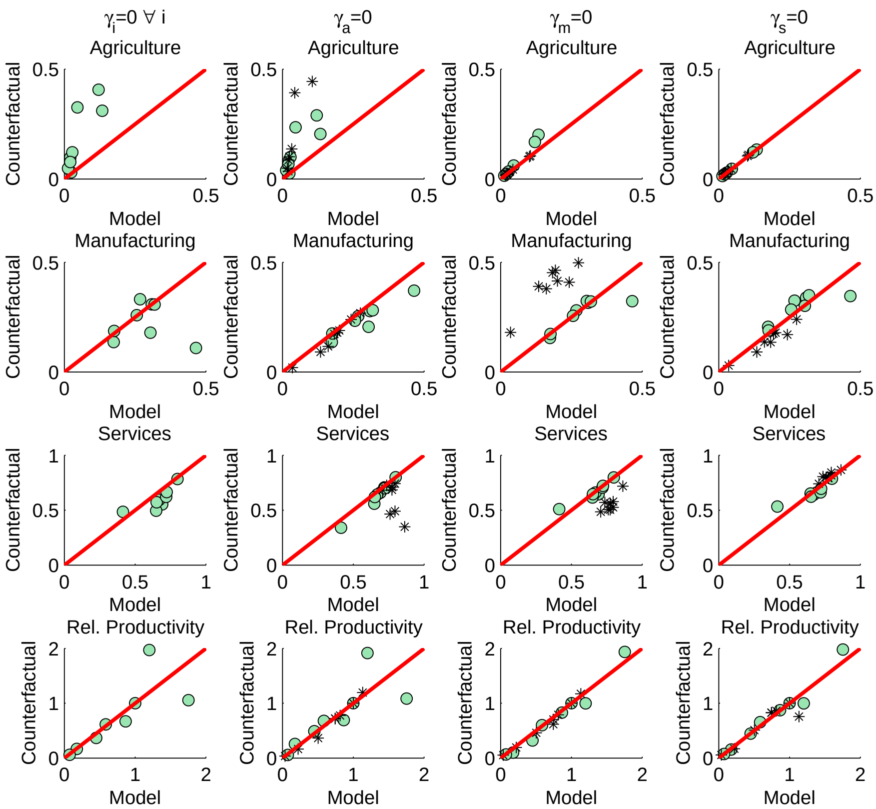

Figure 7.1: The Effect of Zero Productivity Growth

Column one sets all sectoral productivity growth rates (γ_i) equal to zero. Columns two to four set the growth rate of sectoral productivity in a sector to zero in all countries, leaving the other sectors as in the data, for agriculture (second column), manufacturing (third column), and services (fourth column). Each column plots the sectoral labor share and relative aggregate productivity in the end of the sample for the baseline calibration and the counterfactual exercise.

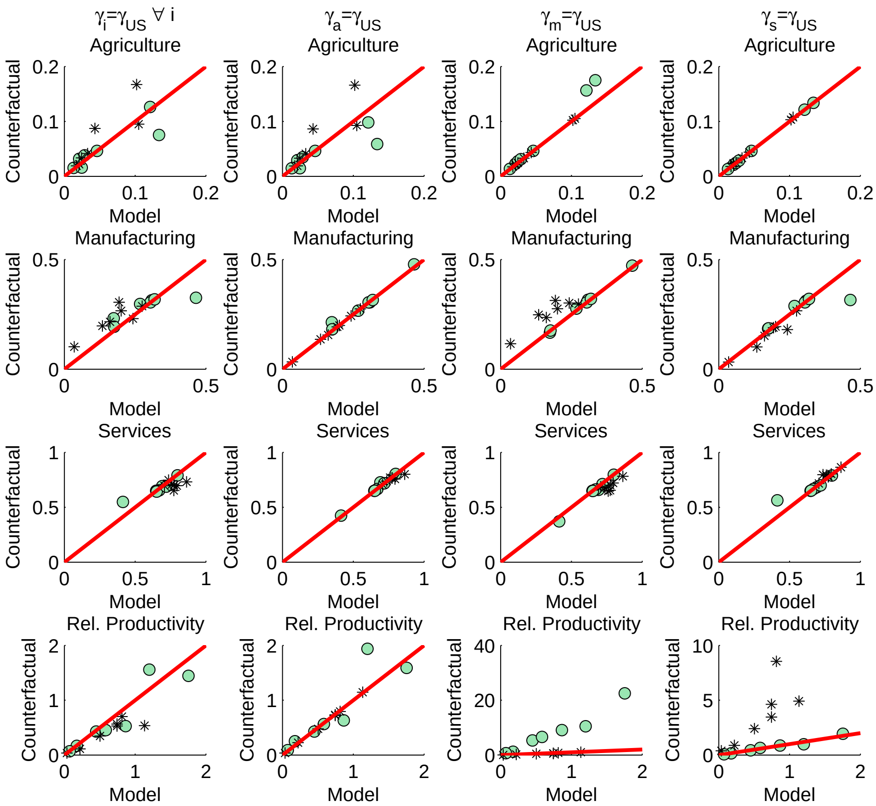

Figure 7.2: Setting Sectoral Productivity Growth Equal to the United States

Column one sets all sectoral productivity growth rates (γ_i) equal to the growth rate of the United States. Columns two to four equalize the growth rate of sectoral productivity in a sector to the U.S. in all countries, leaving the other sectors as in the data, for agriculture (second column), manufacturing (third column), and services (fourth column). Each column plots the sectoral labor share and relative aggregate productivity in the end of the sample for the baseline calibration and the counterfactual exercise.